Stochastic theory of enzyme networks: finite copy number corrections to rate equation models

Abstract

Chemical reactions inside cells occur in very small compartment volumes, typically in the range of atto- to femtolitres. Physiological concentrations realized in such small volumes imply low copy numbers of interacting molecules with the consequence of considerable fluctuations in the concentrations. In contrast, rate equation models are based on the implicit assumption of the very large number of interacting molecules, or equivalently, that reactions occur in infinitely large volumes. In this contribution we compute the finite-volume corrections to the solutions of the rate equations for chemical reaction networks. This is achieved by applying a mesoscopic version of the quasi-steady state assumption to the exact Fokker-Planck equation associated with the Poisson representation of the chemical master equation. The procedure yields impressively simple and compact analytical expressions for the finite-volume corrections. The theoretical predictions are shown to be in excellent agreement with stochastic simulations.